Investigation of PAPR Reduction using Tone Reservation and Active Constellation Extension

Research Summary

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Contents

Introduction 3

1 Theoretical overview 5
  1.1 OFDM signal model ........................................... 5
  1.2 Upper bound of the oversampling ............................ 6

2 PAPR reduction techniques 8
  2.1 Active Constellation Extension (ACE) ....................... 8
    2.1.1 Barsanti ACE method .................................. 8
  2.2 Tone Reservation (TR) ..................................... 10
    2.2.1 Clipping based TR (TR-C) ............................. 11
    2.2.2 Kernel based TR (TR-K) ............................... 11
    2.2.3 Enlipping based TR (TR-E) ......................... 14
  2.3 The adaptive TR algorithm .................................. 15
  2.4 Lower bound of the PAPR value ............................. 15

3 Simulation results 18
  3.1 The simulation environment ................................. 18
  3.2 Simulation of the TR method ............................... 19
    3.2.1 The clipping ratio .................................. 19
    3.2.2 Constant clipping ratio ............................. 19
    3.2.3 Adaptive clipping ratio ............................. 21
    3.2.4 Bit error ratio ................................... 23
  3.3 Simulation of the ACE method .............................. 25
    3.3.1 Constellation diagrams .............................. 25
    3.3.2 The CCDF curves and PAPR value ..................... 26
    3.3.3 The clipping ratio and the BER curve ............... 27
    3.3.4 The joint TR and ACE application (TRACE) .......... 30

4 Measurement results 32
  4.1 The measurement environment .............................. 32
4.2 Comparing the simulation and the measurement . . . . . . . . 33

Conclusion 35

Bibliography 38
Introduction

Orthogonal frequency division multiplexing is a commonly known multicarrier modulation scheme for high speed digital communication. It is used in many wired and wireless communication applications, and it is applied in various standards as well (DVB, WLAN, LTE). The usable frequency range is divided into multiple bands, which can be easily disparted using Fast Fourier Transform (FFT). One major drawback of the modulated signal is the high peak-to-average power ratio (PAPR), because it may negatively affect the overall system performance if amplifiers with limited linear range are applied. During my previous studies [7] I investigated the different PAPR reduction techniques including their drawbacks and benefits [4, 10].

One of these methods is tone reservation (TR), which is also a part of the DVB-T2 standard. Two realization of this technique is presented there, namely the Kernel based TR and the Clipping based TR method [9, 6]. During my research I introduced a novel, third variant, called Enlipping based TR which I also investigated regarding the PAPR reduction performance. In the last semester, I performed different simulations investigating these techniques and I compared them from various aspects. There are a lot of parameters, which can influence the performance of these methods. These are the reserved tones, the modulation type, the iteration number, the clipping ratio (CR) and the number of applied carriers, and it would be essential to investigate these methods with other parameters as well.

Each of the above mentioned techniques is strongly dependent on the chosen CR, and however we proposed an optimal value for the clipping ratio, we should also make further investigation about this parameter. While the clipping ratio has an influence on the overall PAPR value, the investigation of how adaptive CR – where the degree of clipping is not predetermined as a constant value, but it dynamically changes for every single symbol – would affect the PAPR distribution, is also a relevant question.

On the basis of our previous research, it can be stated, that the PAPR value cannot be reduced endlessly. Finding this limit would help to define, how good is a particular method in PAPR reduction, and we would have an overall view about, what we can achieve. With linear programming this lower bound can presumably be
calculated. One additional parameter which has to be considered for the algorithms is the maximal allowed power of the reserved positions. Allowing higher values may lead to better performance, but for practical implementation these values must be kept at a reasonable level. Investigating this limit can also lead to promising results. The number of reserved carriers and their position is also a crucial question, which need to be considered wisely, because it can affect the speed and the performance of the transmission as well.

Comparing these result with the other method, which is also the part of the DVB-T2 standard, called Active Constellation Extension (ACE) [1] also need to be done. It is important to investigate, which technique may lead to better performance, and which should be applied under different circumstances. Also the benefits of the joint use of ACE and TR must be investigated as well. Presenting a measurement environment based on software radio allows us to validate the results of the simulations.

The summary is organized as follows. In Chapter 1 a signal model of the OFDM transmission is given, and the statistical properties of the modulated signal is described. After that, in Chapter 2 a brief overview is given about the examined TR and ACE techniques, and in Chapter 3 the simulation results of the different methods are presented, where we also compare the presented TR schemes from a practical point of view. At the end, in Chapter 4 the measurement environment and results are presented. Finally, Chapter 5 concludes the summary.
Chapter 1

Theoretical overview

1.1 OFDM signal model

The basics of the OFDM systems, that the available bandwidth is divided to multiple subcarriers (channels), which can be at the receiver side easily disparted. Due to the increased number of the carriers, the speed of the transmission also increases.

There are orthogonal, independent subcarriers in the OFDM systems, and the modulation and demodulation can be easily realized with a simple $\mathcal{N}$-point FFT (IFFT), which is basically a summation of modulated complex harmonics, and can be expressed as follows:

$$x_n = \frac{1}{\sqrt{LN}} \sum_{k=0}^{LN-1} X_k e^{j2\pi \frac{kn}{LN}}, \quad 0 \leq n < LN, \quad (1.1)$$

where the index of subcarriers is denoted by $k$, the oversampling factor is denoted by $L$, while the index of the discrete sample and the complex modulation value of the $k^{th}$ subcarrier is $n$ and $X_k$ respectively. Calculating the value of the PAPR for OFDM symbols, we can come to the following expression:

$$\text{PAPR} = \frac{\max \{r_n^2\}}{E\{r_n^2\}} = \frac{\max |x_n|^2}{E\{|x_n|^2\}}, \quad 0 \leq n < N, \quad (1.2)$$

With this formula one can compute the probability, that the PAPR of an N sample long OFDM symbol exceeds a previously determined PAPR0 threshold as

$$P(\text{PAPR} > \text{PAPR}_0) = 1 - (1 - e^{\text{PAPR}_0})^N \quad (1.3)$$

One major drawback of the multicarrier modulation scheme, that the transmission signal has a high PAPR value, which denotes the Peak-to-Avarage-Power-Ratio.
Increasing the number of samples the probability also increases, that the PAPR value exceeds a predefined $\text{PAPR}_0$ value. The high PAPR value negatively effects the overall performance, because then the transmission signal has high peaks, although with low probability. Due to these peaks, the amplifier of the transmitter side will be operating in its nonlinear range, and this makes the transmission worse, and causes distortion [2]. Our goal is therefore to achieve as low PAPR values as we can.

The Complementary Cumulative Distribution Function (CCDF) is often used to visually represent a random variable, namely the PAPR value, when the $\text{PAPR}_0$ value is known. This function can be expressed as follows:

$$P(\text{PAPR} > \text{PAPR}_0) = 1 - P(\text{PAPR}_0)$$ (1.4)

1.2 Upper bound of the oversampling

In order to approximate the real, continuous signal even better, oversampling is often used in practice. For this, we have to define a so-called oversampling factor. It is essential to examine, that how the CCDF curve of the signal changes using oversampling. Due to the increase of the samples, clearly the probability will also higher, that the signal exceeds a certain PAPR value, so the CCDF curve will be shifted to the right, which means that the PAPR value of the signal will be higher [11].

An interesting question, whether this shift has an upper bound, or not. Let us denote the oversampling factor with $L$, where $L > \frac{\pi}{\sqrt{2}}$, and $LN$ is an integer, an and for sufficiently large values of $N$, we can give an upper bound for the CCDF curve of the PAPR value:

$$P(\text{PAPR} > \text{PAPR}_0) < LN e^{-\lambda \left(1 - \frac{\pi^2}{2L^2}\right)}$$ (1.5)

where $N$ means the number of subcarriers. This upper bound of the CCDF curve is presented in 1.1. figure.
Figure 1.1. The upper bound of the CCDF [11], $N=1024$, $L=4$. 

\[ P(\text{PAPR} > \text{PAPR}_0) \]
Chapter 2

PAPR reduction techniques

Multiple methods are known to reduce the PAPR value, and each of them has its own advantages and disadvantages. It is important to examine, whether the techniques have distortion, decrease in the speed of the transmission, bit-error ratio (BER) increase, power increase, or is it necessary to transmit additional information along with the data.

2.1 Active Constellation Extension (ACE)

Applying the ACE method, we extend our external constellation points dynamically so, that the overall PAPR value of the data block should be decreased. There are 2 realizations of this technique, the Krongold and the Barsanti method. The basic idea of the ACE method can be presented easily with the 4-QAM modulation. Each subcarrier can contain one of the 4 constellation points, which are situated in the 4 quadrant of the complex coordinate system. During the technique, each point can be extended to 4 different direction, but we allow only 1, because the other 3 could increase the BER value. The method is presented in figure 2.1. Realisations of the method can be found in [1, 8].

This method can be applied to M-QAM and M-QPSK modulations as well. It is possible the joint usage of the ACE and the later presented Tone Reservation technique. Next I present the main steps for the Barsanti ACE method.

2.1.1 Barsanti ACE method

The Barsanti ACE method has 3 main steps:

- 1. step: Let us form the $X_c$ signal. We can get this by using an IFFT on the original, $X$ signal, and therefore we get the $x$ signal. After that, we make a
clipping with a $V_{\text{clip}}$ threshold on $x$, and so we can get the $x_c$ signal. Applying an FFT on this signal results in $X_c$.

• 2. step: Let us form the $X'$ signal, which can be calculated as follows:

$$X' = G \cdot X_c,$$  \hspace{1cm} (2.1)

where $G$ means the gain, and can be calculated with the following formula:

$$\alpha = 1 - e^{-\gamma^2} + \frac{\sqrt{\pi}}{2^{\gamma \text{erfc}(\gamma)}}$$  \hspace{1cm} (2.2)

$$G = \frac{1}{\alpha},$$  \hspace{1cm} (2.3)
where erfc Gauss error function.

- 3. step: Let us apply an L saturation level as a limit to the extended $X'$ signal, which results in $X''$. Then we choose for $X_{\text{ace}}$ the $X''$ signal, if it fits to the extension criteria, namely its sign is equal as the sign of the $X$ signal, and its magnitude exceeds the magnitude of $X$. Otherwise we choose for $X_{\text{ace}}$ the $X$ signal. At the end, we apply an IFFT on the $X_{\text{ace}}$ and then we get the $x_{\text{ace}}$ signal.

The block diagram of the Barsanti ACE method is presented in figure 2.2.

![Block diagram of Barsanti ACE method](image)

**Figure 2.2.** Block diagram of Barsanti ACE method

### 2.2 Tone Reservation (TR)

Tone Reservation is an effective way to reduce the PAPR value of multicarrier signals. This method is based on a principle, where we keep pre-determined subcarriers, where no usable data is transmitted, only an auxiliary signal, which is added to the main signal helps to reduce the PAPR value. This signal reduce the peaks in the original signal, and it can be easily computed in the transmitter side, and removing in the receiver side is also really simple. There are 2 known methods of Tone Reservation, which are the part of the DVB-T2 standard, the Kernel and the Clipping based TR [10, 4]. An other, novel variant, called Enlipping based TR is also introduced.
2.2.1 Clipping based TR (TR-C)

In this method, first we perform a clipping on the $x_n$ signal. Here $x_n$ represents the $n^{th}$ element of the $x$ vector. After that the clipped signal, $y_n$, is subtracted from the original, input $x_n$ signal, which results in a so-called correction $c_n$. On this signal, we apply an FFT function.

$$c_n^i = x_n^i - y_n^i \quad (2.4)$$

$$C_k = \text{FFT}(c_n) \quad (2.5)$$

To fulfill the requirements of the TR method, we have to keep the values only on the pre-determined positions, which are the Peak Reduction Carrier (PRC) positions. The other values are set to 0.

$$\hat{C}_k = \begin{cases} C_k & k \in \text{PRC}, \\ 0 & k \notin \text{PRC}. \end{cases} \quad (2.6)$$

Every iteration results in a $\hat{C}_k$ vector, and applying an IFFT on that signal, we get a time-domain signal $\hat{c}_n$. At the end of each iteration this clipped and filtered correction signal is added to the $x_n^i$ input vector.

$$x_n^{i+1} = x_n^i + \mu \hat{c}_n \quad (2.7)$$

The iteration number, and the value of $\mu$, which denotes the value of the weight factor, are the parameters of the technique. Our job is to find the optimum values of these parameters. It can be easily seen, that only the values of the PRC position had been altered, therefore there is no distortion in the transmission, although the PAPR value will be lower. The block diagram of the TR-C method is shown in figure 2.3..

2.2.2 Kernel based TR (TR-K)

The Kernel based TR can be divided into 2 main steps. These are the followings:

- **1. step**: We create a kernel vector, which denoted by $p_n$. To achieve an optimal result, this generated kernel vector should be a discrete, impulse-like function.
Ideally it should be one peak, but in real life this cannot be realized, so the kernel vector is defined by the following expression:

$$p_n = \sqrt{\frac{N_{FFT}}{N_{PRC}}} \text{IFFT}(P_k), \quad (2.8)$$

where $N_{FFT}$ means the FFT size and $N_{PRC}$ denoted the number of the PRCs. The $P_k$ vector is $N_{FFT}$ size vector, where at the PRC positions there are ones, and every other values are zeros. A circularly shifted kernel can be seen in figure 2.4.

- 2. step: During the peak reduction algorithm, we perform an IFFT on the frequency domain signal $X$, which results in $x$. In every iteration we have to find the maximum amplitude and its position of the $NL$ values:

$$A^i = \max_{n} |x^i_n|, \quad (2.9)$$

$$m^i = \arg \max_{n} |x^i_n|, \quad (2.10)$$

where $x^i_n$ means the $n^{th}$ element of the $x^i$ vector, $A^i$ and $m^i$ is the maximum amplitude and its position during the $i^{th}$ respectively. An OFDM symbol is presented in figure 2.5., where the maximum amplitude is indicated.

After these steps we have to circularly shift the kernel vector so, that the maximal amplitude of $p_n$ will be at the same position as $m^i$, and we scale and phase rotate it so, that the power of the peak should be reduced to a previously determined target clipping level. The modified kernel is subtracted
Figure 2.4. Circularly shifted kernel according to the DVB-T2 standard, $N = 1024$

from the original $\mathbf{x}$ vector, and then we calculate the PAPR value. These steps can be described as follows:

$$x^{i+1} = x^i - \alpha^i p_n(m^i),$$  \hspace{1cm} (2.11)

$$\alpha^i = \frac{x^i(m^i)}{A^i} (A^i - A_{\text{max}}),$$  \hspace{1cm} (2.12)

where $p_n(m^i)$ means the circularly shifted kernel, while $A_{\text{max}}$ denotes the clipping amplitude. If the calculated PAPR value reaches the previously determined level, the algorithm is terminated. Otherwise the second step has to be repeated. If an iteration number is exceeded, then the method also has to be stopped. We have to previously determine both the PAPR threshold and the iteration number. The transmission signal after the $i^{th}$ iteration can be
expressed as follows:

\[ x^i = x + \alpha^1 p_n(m^1) + \ldots + \alpha^i p_n(m^i) = x + \sum_{k=1}^{i} \alpha^k p_n(m^k). \] (2.13)

The block diagram of the TR-K method is presented in figure 2.6.

### 2.2.3 Enlipping based TR (TR-E)

The TR-E method is a novel, clipping based technique, which was originally introduced with the ACE method [12]. I applied the main concept of this algorithm to the TR method. The TR-E is very similar to TR-C, the only difference is that we did not only perform a clipping on the carriers, which exceed the clipping level, but we set all amplitude of the subcarriers to the predefined \( A_{\text{max}} \) level. The resulting signal is very similar to the one we got during the TR-C algorithm:
Figure 2.6. Block diagram of the kernel based TR

\[ y_n = A_{\text{max}}e^{j\phi(x_n)} , \]  

where \( A_{\text{max}} \) is the clipping amplitude. The following steps are identical to the ones, we presented at the TR-C method. With the help of the TR-E technique, we can reach lower PAPR values, than with the TR-C.

### 2.3 The adaptive TR algorithm

Defining the right CR value for each, presented TR algorithm is a crucial question. As a new idea, instead of defining a constant CR, we dynamically changes it at every iteration. Simulating every case, we can determine which clipping trajectory is the most optimal, and later, we can use it instead of a constant CR value. The idea was verified with simulations, and the results are presented in the next chapter.

### 2.4 Lower bound of the PAPR value

To find the lower bound of the PAPR value, which can be achieved with the TR method, we have to make a mathematical model. After defining the problem, we have to choose the proper mathematical method and model, and we have to define its parameters as well. This is followed by the solving of the model, and after the practical realization, we can check, whether our result satisfies our conditions, or not.
We have to define a target function, which value has to be minimized or maximized. The word linear refers to the fact, that every function in our model is linear. The lower bound of the PAPR value for ACE summarized in [5], and it can be easily applied to the TR method with a few modifications.

Defining the target function for the PAPR reduction means we have to find the minimal PAPR value, depending on which values are of the predefined PRC positions. We can find the optimal solution for this problem besides the given conditions by writing up the problem in an exact form. First we have to express the modified signal as the sum of the original OFDM symbol and an auxiliary signal:

\[
\tilde{x} = x + r
\]  

(2.15)

Based on this definition, we are looking for the signal \( r \) which value has to be changed to get to the optimal signal, and therefore to the optimal PAPR value. The frequency domain representation \( \tilde{X} \) of the resulting \( \tilde{x} \) signal has to be fulfill the conditions of the TR method, which means that the signal at the PRC positions can be chosen freely, but the values of the data subcarriers cannot change their values.

We can formulate the problem generally as follows:

\[
\min E
\]

\[
|x + r| < E
\]  

(2.16)

This means, that we define an upper bound for the \( x + r \) signal for every sample of the symbol, which is denoted by \( E \). With complex numbers, the absolute value of the sample is defined by the root of the sum of the square of the real and imaginary part. So we can say that the problem is Quadratically Constrained Quadratic Program (QCQP). The problem is defined in the discrete time domain, however the criteria are defined in the frequency domain. We can transform between the two domains with the help of the IDFT. We can express this in a linear algebraic format with an \( LN \times LN \) matrix, which is denoted by \( F \), while the opposite direction of the transformation is denoted by \( F^{-1} \). With the help of this matrix, we can express the time domain signal in a form \( x = XF \). During the optimization problem we handle the real and imaginary parts separately, so in the followings we use the presented
notations:

\[ F_{Re} = Re\{F\}, F_{Im} = Im\{F\}, \quad (2.17) \]
\[ X_{Re} = Re\{X\}, X_{Im} = Im\{X\}, \quad (2.18) \]
\[ R_{Re} = Re\{R\}, R_{Im} = Im\{R\}, \quad (2.19) \]

where \( Re \) and \( Im \) are the operators for the real and imaginary parts. Let us denote furthermore

\[ x_{Re} = F_{Re}X_{Re} - F_{Im}X_{Im} \quad (2.20) \]
\[ x_{Im} = F_{Re}X_{Im} - F_{Im}X_{Re} \quad (2.21) \]

the real and imaginary parts of the samples of the discrete signal in the time domain. We use only a small set of the subcarriers to reduce the PAPR value, therefore the \( F \) matrix has a size of \( M \times LN \), where \( M \) are the numbers of the PRCs, and \( N \) denotes the number of the subcarriers, while \( L \) is the oversampling factor. The problem, which leads to the optimal solution for the TR algorithm, is expressed by the following equations:

\[
\begin{align*}
\min E \\
\left[ \begin{array}{cccc}
F_{Re} & -F_{Im} & x_{Re} & 0 \\
F_{Im} & F_{Re} & 0 & x_{Im}
\end{array} \right] \begin{bmatrix} R_{Re} \\
R_{Im}
\end{bmatrix}
+ \begin{bmatrix} 1 \\
1
\end{bmatrix}
\begin{bmatrix} E_{Re} \\
E_{Im}
\end{bmatrix}
= \begin{bmatrix} 0 \\
0
\end{bmatrix} \\
\end{align*}
\]

\[ \begin{align*} \\
-\infty < R_{Re} < \infty \\
-\infty < R_{Im} < \infty \\
E \geq \sqrt{E_{Re,k}^2 + E_{Im,k}^2}
\end{align*} \quad (2.23) \]

With the aid of the method, we can determine the real and imaginary parts of the signal \( r \), which will be placed at the PRC positions, and which results in a minimum PAPR value. For this technique, and for determining the optimal solution I used the MOSEK7 toolbox of the Matlab simulation environment.
Chapter 3

Simulation results

3.1 The simulation environment

The PAPR reduction capability of the presented 3 TR method were verified through simulations in the Matlab simulation environment. Every block of the simulator were created individually, which resulted in an easily expandable, modular system. The blocks of the transceiver and the receiver are shown in figure 3.1. and figure 3.2..

Figure 3.1. Block diagram of the transceiver

Figure 3.2. Block diagram of the channel and the receiver
3.2 Simulation of the TR method

With the unique simulation environment one can easily set every parameter of the system, and these parameters are summarized in table 3.1. and are based on the DVB-T2 standard. The investigations can be expanded later to other set of parameters as well.

<table>
<thead>
<tr>
<th>Name of the Parameter</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>FFT size</td>
<td>1024</td>
</tr>
<tr>
<td>Number of PRCs</td>
<td>10</td>
</tr>
<tr>
<td>Position of PRCs</td>
<td>116, 130, 134, 157, 182, 256, 346, 478, 479, 532</td>
</tr>
<tr>
<td>Maximal allowed amplitude</td>
<td>5</td>
</tr>
<tr>
<td>Modulation</td>
<td>4-QAM</td>
</tr>
<tr>
<td>Number of carriers (N)</td>
<td>1024</td>
</tr>
<tr>
<td>Number of symbols</td>
<td>15000</td>
</tr>
<tr>
<td>OV factor</td>
<td>4</td>
</tr>
<tr>
<td>Value of $\mu$ at the TR-C and TR-E methods</td>
<td>-1</td>
</tr>
</tbody>
</table>

3.2.1 The clipping ratio

As I mentioned before, the clipping ratio (CR) is a really important parameter of the PAPR reduction, because at different CR values different PAPR reductions can be observed. There are no references to the examination of this value at previous studies, therefore the simulations first have to determine, how the amount of the PAPR reduction is depend on the value of the CR. There are two scenarios, which has to be investigated.

3.2.2 Constant clipping ratio

During the iterations we define a constant clipping ratio. In figure 3.3. can be observed, how the PAPR value at $10^{-1.8}$ probability with different CR values and iteration numbers changes at the 3 method. At lower probability levels the simulated CCDF curve is not reliable. For lower values further and longer simulations are required. Some of the results below summarized in my previous work [7].

The CCDF curves of the different techniques are shown in figure 3.4.. Those CR values were chosen for each methods, which leads to the best performance at the PAPR value. At the figure the theoretical, reachable minimum is also indicated, which was determined with the previously presented model.

Every presented TR method lead to better PAPR values, as the method without PAPR reduction. The TR-K technique has the best performance, and it outperforms the others in complexity, because the algorithm works only in the time domain. As
we can see, the Enlipping based TR method is better at convergence steps not just the TR-C, but the TR-K technique as well. It also has a better performance at PAPR reduction as the TR-C. However its computational complexity is higher, than the one that TR-K has, due to the extra IFFT/FFT operation, which has to be performed at each iteration. It can be stated, that the TR-C or the TR-E method has to be used in those cases, when a kernel function is not available, or cannot be generated.

During the investigations we used the values of the DVB-T2 standard regarding the PRC positions. It would be also relevant to examine other PRC positions as well, but as the standard states, determining the PRC positions is the result of a long, optimization process [4]. If the positions of the reserved tones are random, the it is recommended to use the Enlipping based TR technique. Higher PRC number
results in better performance at the TR-C and TR-E method, but then the speed of the transmission is also reduced, because these tones do not carries any information. The allocated maximal energy at the carriers is also an important parameter of the simulations. Allowing higher values is not recommended for practical implementation, so defining this parameter is also an essential question. However the simulations show, that the TR-K and TR-C method is insensitive for this parameter, although the TR-E is highly dependent from its value. Determining the value of the maximal allowed energy is the subject for further examinations.

3.2.3 Adaptive clipping ratio

As mentioned before, it is also important to investigate the possibility to apply an adaptive clipping ratio. During the simulations the same parameters were used, as before, the only difference is that the CR was not a constant value, but it changed at every iteration. I investigated the PAPR value for 3 iteration, and at each iteration
we set the value of the CR to an exact value between 1 and 8, and every variations were examined. We looked for the minimum spot, an let us say, that the first CR is constant, and the second and third CR is presented on the two axis of the coordinate system. Figure 3.5. shows the optimal adaptive clipping ratio for the 3 method, the minimum spots are indicated with a white cross.

![Graph showing adaptive clipping ratios and PAPR values](image)

(a) TR-C, $CR_{1,2,3} = 2\, dB, 4\, dB, 5\, dB$, $PAPR = 10.765\, dB$

(b) TR-K, $CR_{1,2,3} = 6\, dB, 7\, dB, 7\, dB$, $PAPR = 9.273\, dB$

(c) TR-E, $CR_{1,2,3} = 8\, dB, 7\, dB, 8\, dB$, $PAPR = 10.458\, dB$

**Figure 3.5.** The achieved suboptimal PAPR values with adaptive clipping

It can be seen, that with the TR-K and TR-C method the PAPR values are better, than the values with the constant CR. The Enlipping based TR technique shows no improvement, because its convergence is really fast, and its performance is not so dependent from the CR value. The adaptive clipping ration results in an amount of 2.36% improvement for the TR-K and an amount of 17.79% improvement for the TR-C. At higher iteration numbers the improvement of the performance is probably even higher.
Figure 3.6. The achieved suboptimal PAPR values in magnified resolution

Close to the minimum, I performed further simulations to achieve better results about the local minimum spot. These results are shown in figure 3.6. It can be stated, that these investigations do not improve the PAPR value significantly, it only makes the computational complexity higher. After these examinations an optimal clipping profile can be created, and we can make further improvements regarding the PAPR value with it.

3.2.4 Bit error ratio

During the simulations of the bit error ratio multiple things had to be taken into consideration. Due to the different algorithms, the maximal amplitude of the subcarriers has been reduced, so the signal can be amplified. After that is was essential to investigate, that how the bit error ratio changes at different signal-to-noise ra-
ratio values (SNR). The additive white gaussian noise (AWGN), which is added to the signal has to be equal to the one, which belongs to the original input signal. At the receiver side, the signal is downsampled, and after the inverse mapping, we compared the received bits to the transmitted ones. The result of the simulation is presented in figure 3.7.

![Figure 3.7. Bit error ratio at the different methods, N = 1024](image)

We can observe, that the almost all of the methods improves the bit error ratio. It was expected, because due to the PAPR reduction, the signal could be amplified more, so its tolerance against the environmental noise is increased. The optimal solution has the best improvement at the BER, and after that comes the TR-K and the TR-C method. However the optimal solution cannot be applied in real-time applications, because it has a high complexity, while its execution time is really long. The TR-E technique makes the bit error ratio a little bit worse. It happens due to the fact, that this method probably reduces the PAPR value not because it reduces the peaks, but because it brings energy to the system, which means that it makes the amplitudes of the other carriers higher. It can be stated, that the method, which was originally presented along with the ACE algorithm [12], is not applicable at every situation for the TR algorithm. The improvement of the PAPR is not enough to state that the transmission became better, the value of the bit error ratio is also
has to be investigated. It could be essential to examine the maximal amplitude of the carriers and the value of the weight factor \( \mu \) too, because it may lead to a case, when the Enlipping based TR method can reduce the PAPR value and the bit error ratio at the same time.

It is also important to examine the BER values at the adaptive clipping ratio as well. The improvement of the TR-K method at the PAPR values is not so significant, therefore the improvement at the BER curve can not be easily noticed, but due to the better results of the TR-C technique, we can see the same improvement at BER curve as well.

### 3.3 Simulation of the ACE method

During the previous simulation in the literature [5, 8], the parameters - the number of subcarriers or the iteration number - were always a constant value. Nevertheless I wanted to examine these parameters and the correlation between them a little bit deeper. During the simulations I dealt with different subcarrier numbers, but at the presented result it is always 1024, because my previous result used the same value.

#### 3.3.1 Constellation diagrams

The parameters of the simulations for the ACE method is summarized in table 3.2.

<table>
<thead>
<tr>
<th>Name of the parameter</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>FFT size</td>
<td>1024</td>
</tr>
<tr>
<td>Maximum allowed amplitude</td>
<td>5</td>
</tr>
<tr>
<td>Modulation</td>
<td>4-QAM</td>
</tr>
<tr>
<td>Number of carriers (N)</td>
<td>1024</td>
</tr>
<tr>
<td>Number of symbols</td>
<td>5000</td>
</tr>
<tr>
<td>OV factor</td>
<td>4</td>
</tr>
</tbody>
</table>

During the examinations I investigated how a clipping alters the positions of the constellation points. These points are scattered near to 4 point in the case of 4-QAM modulation. We do not want to worsen the bit error ratio, therefore the extension is only allowed in that one direction, which makes no result in the BER value. The point in the other 3 directions is taken back to the original point. The constellation extension is presented in figure 3.8..

If the transmission of the OFDM symbol is realized, then the signal has to go through a noisy channel. The additive noise makes the scattering of the constellation points even larger, which results in worse BER values.

The modified version of the ACE is called ACEPro. Its main step, that the points, which are not scattered in the right direction, are not taken back to the original
point, they are taken back only into a position, which is in the same line, as the original point. Therefore the extension can be applied to more points, which results in further PAPR reduction. The constellation diagram of the ACEPro is shown in figure 3.9.

It is important to say a few words about limits of the method. The results of the algorithm are surprisingly good, but it not so well applicable at higher constellations. Let us say, that we use 4-QAM modulation, then we have for 4 point 4 direction to extend the points. At a 1024-QAM modulation this number is much smaller, there are only 126 points, which even has an extension direction, and only the 4 corner point has a large extension area.

3.3.2 The CCDF curves and PAPR value

First the PAPR reduction performance of the ACE and ACEPro algorithms were investigated. Both algorithm is iterative, and we can see, that increasing the iteration number the PAPR reduction is also increases. It is also important to examine, how the ACE and the TR algorithm relate to each other. The simulations were performed with the best TR method, namely with the TR-K with 10 iteration and with the optimal CR=6.9 dB value, but it can also be performed with the TR-C or with
the TR-E easily. As we can see in figure 3.10a., the ACEPro outperforms the ACE method, and it is presented in figure 3.10b., that if the ACE method is performed with 1 iteration, then the TR-K has better results, but if the ACE is executed with 3 or 5 iterations, then the constellation extension has the better PAPR reduction performance.

3.3.3 The clipping ratio and the BER curve

It is important to determine an optimal clipping ratio to the ACE method as well. I investigated this with the previously presented algorithm, namely I executed the algorithm with different CR values, and I investigated how the PAPR value changes at $10^{-1.8}$ probability. I performed the algorithm with 1, 3, 5 and 10 iteration. The result are shown in figure 3.11.

We can see at the curves, that with increasing iteration number the amount of the PAPR reduction also increases, and the ACE algorithm has its maximal PAPR reduction at 1 dB. This means a really high clipping ratio, because as we know, 0 dB

![Figure 3.9. Modified constellation diagram with 4-QAM, N=1024](image)
means that our clipping factor is 1, which results in a clipping level, that is around the root of the signals’ mean power.

To determine the results, which can be achieved with this CR, we have to examine the BER curves. The basics of the simulations was the same, as previously, which means I amplified the signal, and then I added to it the original noise. The simulations were performed to 3, 5 and 10 iterations as well, and at each clipping ratio I represented the BER curves. The result are summarized in figures 3.12.
Figure 3.12. BER curves of the ACE method

(a) BER curves for different CR values, iteration=3

(b) BER curves for different CR values, iteration=5

(c) BER curves for different CR values, iteration=10

The first observation is that the optimal CR is not 1 dB, but 3 dB. We can see, that to investigate only the PAPR values and the CCDF curves is often not enough. It can be seen best in figure 3.12c., that the 0 dB clipping ratio does not result in better BER values, it sometimes make the bit error ratio even worse. We can get to similar conclusions with the ACEPro method too.

To have a deeper understanding of the problem, I examined the constellation diagram of the ACE and ACEPro methods with 0, 1 and 3 dB CR. The results of the simulations are shown in figures 3.13..

At lower clipping values, which means higher clipping level, the constellation points scatter farther, which are presented at the figures. The higher scattering makes the bit energy larger, which means, that we have to use more energy to transmit these points, and therefore we cannot amplify our signal the way we want, which results in worse BER values. As a conclusion, we can say, that with higher
CR better PAPR values can be achieved, but it is also important to hold the scattering in hand. This can be done with the joint investigation of the BER and the constellation diagrams, and we can also introduce a saturation level. We can see in figure 3.13a. the maximal amplitude, what we previously set to 5.

3.3.4 The joint TR and ACE application (TRACE)

During the simulations I executed the TR-K and the ACE algorithms after each other. The parameters of TR-K method is the same, as mentioned before, and the CR for the ACE technique was set to the previously calculated 3 dB. First the
constellation extension was performed, and after that the TR-K algorithm was executed. It is important, because we did not want to ruin the signal at the PRC positions with the ACE algorithm. After the execution of the ACE method, the PRC positions were set to zeros. The results of the simulations are shown in figure 3.14.

The results show, that if the ACE algorithm is performed with the iteration number set to 5 or 10, then the PAPR value of the signal, and therefore the peaks are reduced so much, that TR-K method will not be executed, and because we set the PRC positions to zero, a few subcarriers will disappear. This will increase the PAPR value, because we removed some of the signal, which had a part in the PAPR reduction. With iteration number 3, the TR-K technique will be performed, and the joint application of the 2 methods results in a better PAPR value, than the single ACE or TR-K method. The TRACE has the best performance, when the CR for the TR-K is around 5.5 dB. We can see, that this value is different from the previously determined optimal value, which was 6.9 dB. The TRACE algorithm should be used at larger constellations, and when the iteration number due to the speed of the transmission is limited.
Chapter 4

Measurement results

During the measurements I transmit a signal, which was generated with a computer, and with receiving it, I could make some statements about the channel. At the end of the measurements I calculated the PAPR value, and I compared this value with the one, that I was generated during the simulation, and therefore I validated the simulation environment.

4.1 The measurement environment

The measurement was executed with the aid of a Software Defined Radio (SDR). The SDR is a communication system, where certain components, which are usually realized in hardware (e.g. mixers, filters, amplifiers, modulators/demodulator), are implemented in software, mostly at a PC or at an embedded system. In a simple SDR system, the large amount of the signal processing of the special hardware elements are executed by the general purpose microprocessor. SDR systems are widely used at the military and in the telecommunication as well. I used to the measurements an SDR, which was made by Ettus Resarch. The USRO N210 is shown in figure 4.1..

The software radio (URSP) was connected to the PC with an Ethernet cable, in which the the oversampled signal data was transmitted, and at the same channel we could receive the output signal as well. The attenuation between the Rx and Tx input and output was 20 dB, and they were linked with a cable. To the measurement I used the GNURadio software under Linux operation system. GNURadio is a free software, which is able to execute different signal processing algorithms, and with the GUI the user can easily create simple software radio of signal processing systems. After reading the data from the file, I set an attenuation to the system, because I normalized previously the amplitude of the subcarriers. The parameters of the measurement are the same as the parameters of the simulation. The block diagram of the transmission is shown in figure 4.2..
4.2 Comparing the simulation and the measurement

The figures of the measurement shows, that the results of the simulation and the measurement are equal. There is a little difference between the CCDF curves of the simulation and measurement of the optimal solution and the TR-K method. This difference can be explained with the fact, that the simulation SNR parameters are
not the same as the SNR parameters in the measurement environment. The Power Spectral Density (PSD) figures of the different techniques show, that there is no Inter Symbol Interference (ISI) by the symbols, and we can easily recognize the PRC positions. It can be stated, that our simulation model reflects the reality quite good. The measurement and simulation results are summarized in figures figrefpaprmmessim and 4.4. The difference at the PSD figures are caused by the Cascaded Integrator Comb (CIC) filters of the USRP.

(a) The CCDF curves of the measured symbols
(b) The CCDF curves of the simulated symbols

Figure 4.3. Comparison of measurement and simulation

(a) The PSD curves of the measured symbols
(b) The PSD curves of the simulated symbols

Figure 4.4. Comparison of measurement and simulation
Conclusion

In my research I investigated the Active Constellation Extension and the Tone Reservation PAPR reduction techniques. First, I presented the main parameters of the OFDM systems, then one ACE [1] and two TR methods [10] were introduced. I presented a novel, Enlipping based TR algorithm. All of the methods is highly dependent from the clipping ratio (CR), so the investigation of this parameter was a major question of my research, because there are no other research regarding the CR in the literature. A novel, adaptive CR technique was also introduced, which based on the fact, that the CR can be altered during the iterations. Determining the optimal, maximal achievable PAPR reduction with the TR method is a good measure of the performance of the algorithms, and it can be computed with the aid of the Matlab simulation environment.

To investigate the TR and ACE methods, I developed a simulator in Matlab, in which the parts of the transmitter and the receiver were realized by blocks, and therefore the system could be easily extended. I performed simulations about the presented ACE and TR techniques regarding the PAPR values, the CR values, the constellation points, the CCDF, the BER and the PSD curves. The optimal CR values were determined for each method, and it was shown, that the adaptive clipping ratio can make the algorithms even better. Every method is highly dependent also from the iteration number, but for practical implementation this value has to be kept at a reasonable level. The TR-K method outperforms the TR-C and TR-E methods, while at smaller constellations the ACE and ACEPro leads to better results, than the Kernel based TR algorithm. However for larger constellations, the PAPR reduction capability of the ACE methods becomes worse. In this case a joint TR and ACE (TRACE) application could be suitable, but applying the two methods at once leads to different optimal CR values, as before. We have showed, that the examination of the BER curve is often essential, just like the investigation of the constellation layout.

At the end of my research, I presented the measurement environment, which helped me to verify the simulation results, and showed, that the simulation are operating correctly in real life too. The examination of these systems are not a
straight forward task, we have to take many circumstances and parameters into consideration and we have to examine the whole system from different aspects. Deeper investigation of the parameter $\mu$, the maximum allowed power and the TR-E method is part of my future work. The Cubic Metric (CM) reduction [3] method could also give us a better understanding of these systems and parameters, and therefore I would also like to get a deeper sight into the theory of the CM, and I would also like to implement these algorithms in the future for CM reduction.
Bibliography


[4] ETSI. Digital video broadcasting (dvb); frame structure channel coding and modulation for a second generation digital terrestrial television broadcasting system (dvb-t2). ETSI EN 302 755 V.1.3.1, April 2012.


